Inequality and the Zero Lower Bound

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Outline

1 Introduction

2 Model

3 Solution Approach

4 Results: Aggregate Dynamics and IRFs

5 Inflation Target and Real Interest Rates

6 Conclusion

Motivation

- Secular decline in global real rates over the past 30 years.
  Fiorentini, Galesi, Perez-Quiros, and Sentana (2019), Del Negro, Giannone, Giannoni, and Tambalotti (2019)
- Decline made acuter by the 2007-2008 financial crisis and the COVID-19 shock.
- As a result, the zero lower bound (ZLB) on nominal rates has become a pervasive feature of advanced economies.
- Traditional analysis of the macro effects of the ZLB rely on representative agent models.
  Eggertsson and Woodford (2003), Christiano, Eichenbaum, and Rebelo (2011), Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2015)

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- In this paper, we argue that the effects of the ZLB on both aggregate dynamics and the stance of monetary policy crucially depend on household inequality.

What Do We Do?

- Heterogeneous-agent new Keynesian (HANK) model with aggregate shocks and the ZLB.
  - Fully non-linear solution: neural networks approximate the aggregate laws of motion
    Fernández-Villaverde, Hurtado, and Nuño (2020)

- The presence of the ZLB reduces the level of the interest rates through three channels.

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3 Precautionary savings due to aggregate risk:
  - ZLB recessions are relatively larger and weigh more on wealth-poor households.
  - Agents insure against the occurrence of ZLB events.

The Long-run Fisher Equation

- In this setting, monetary policy is non-neutral in the long run.
  - A reduction in the inflation target leads to a drop in the real interest rate.
  - The model features a long-run Fisher equation that equals

$$i(\tilde{\pi}) = r(\tilde{\pi}) + \pi(\tilde{\pi}), \qquad \text{where } dr/d\tilde{\pi} > 0.$$

  - Households' inequality amplifies the degree of non-neutrality.

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  - Households' inequality amplifies the degree of non-neutrality.

- Changes in trend inflation and households' inequality jointly explain 20% of the drop in real rates over the recent decades.
  - We consider a drop in trend inflation from 4% to 1.7% and an increase in the wealth Gini matching its variation in the 2000s.
  - The real rate drops by 46 bps in our HANK economy and 14 bps in the RANK model.
  - In the data the real rate drops by around 150 bps from the late 1980s on

Related Literature

- Representative agent models.
  - **Monetary policy and low rates:** Blanchard, Dell'Ariccia, & Mauro (2010); Andrade, Gali, le Bihan, & Matheron (2019).
  - **ZLB:** Eggertsson & Woodford (2003); Christiano, Eichenbaum, & Rebelo (2011); Fernández-Villaverde, Gordon, Guerrón-Quintana, & Rubio-Ramírez (2015).
  - **Deflationary bias:** Adam & Billi (2007), Nakov (2008), Hills, Nakata, & Schmidt (2019); Bianchi, Melosi, & Rottner (2020).
- Heterogeneous agent models.
  - **Nominal Rigidities:** McKay, Nakamura, & Steinsson (2016); Kaplan, Moll, & Violante (2018); Luetticke (2019).
  - **Methodology:** Krusell & Smith (1998); Boppart, Krusell, & Mitman (2018); Auclert, Bardóczy, Rognlie, & Straub (2019); Fernández-Villaverde, Hurtado, & Nuño (2020).
- Closest papers.
  - **HANK with Permanently-binding ZLB:** McKay & Reis (2016); Auclert & Rognlie (2020).

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Setup

- Discrete-time, infinite horizon, sticky-price economy.
- Heterogeneous households.
  - Ex-ante identical and face idiosyncratic productivity shocks.
  - Choose consumption, bond holdings, and labor supply.
  - Bond holdings limited by a borrowing constraint.
- Firms.
  - Final-good producer (perfectly competitive; CES aggregator).
  - Intermediate-good producers (monopolistic competition).
  - Nominal rigidity: Rotemberg price adjustment costs.
- Preference shocks as source of aggregate uncertainty.
  Christiano, Eichenbaum, and Rebelo (2011)

Households

- Households maximize expected discounted utility

$$\max_{\{c_{i,t},b_{i,t},h_{i,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \frac{1}{1-\sigma} \left(c_{i,t} - \chi \frac{h_{i,t}^{1+\nu}}{1+\nu} \right)^{1-\sigma}$$

$$\text{s.t.} \quad c_{i,t} + b_{i,t} = w_t s_{i,t} h_{i,t} - \tau \left(w_t s_{i,t} h_{i,t} \right)^{1-\gamma} + \frac{R_{t-1}}{\pi_t} b_{i,t-1} + \Pi_t s_{i,t} \,,$$

$$b_{i,t} \geq \underline{b}$$

- Aggregate preference shock ξ_t follows AR(1) process.
- Idiosyncratic productivity shock $s_{i,t}$ follows a Markov chain.
- Progressive labor-income taxation (i.e., flat tax if $\gamma = 0$).
- Bond holdings limited by the borrowing constraint $\underline{b}$.
- Firm profits Π_t are re-distributed according to households' idiosyncratic productivity.

Firms

- Final-good producer assembles intermediate goods with CES function

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

- Production function of intermediate-good producers is $y_{j,t} = l_{j,t}^{\alpha}$.

- Intermediate-good producers choose prices $\{p_{j,t}\}_{t\geq 0}$ to maximize

$$\mathrm{E}_t \sum_{k=t}^{\infty} \beta^k \left[\underbrace{\left(\frac{p_{j,k}}{P_k} - mg\ cost_k \right) \left(\frac{p_{j.k}}{P_k} \right)^{-\varepsilon} Y_k}_{\text{Profits net of adjustment cost}} - \underbrace{\frac{\theta}{2} \left(\log \left(\frac{p_{j,k}}{p_{j,k-1}\tilde{\pi}} \right) \right)^2 Y_k}_{\text{Rotemberg adjustment cost}} \right],$$

where $\tilde{\pi}$ is the inflation target and P_t is the aggregate price level.

- Solving this problem yields the New Keynesian Philips curve

$$\log \left(\frac{\pi_t}{\tilde{\pi}} \right) = \beta E_t \left[\log \left(\frac{\pi_{t+1}}{\tilde{\pi}} \right) \frac{Y_{t+1}}{Y_t} \right] + \frac{\varepsilon}{\theta} \left(mg\ cost_k - \frac{\varepsilon-1}{\varepsilon} \right).$$

Monetary and Fiscal Authority

- The monetary authority follows a Taylor rule subject to the ZLB constraint

$$R_t = \max\left\{1, \tilde{R}\left(\frac{\pi_t}{\tilde{\pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\tilde{Y}}\right)^{\phi_y}\right\},$$

where $\tilde{R}$ is the steady-state nominal rate, and $\tilde{Y}$ is steady-state output

- The fiscal authority raises progressive labor income taxes to finance a fixed amount of outstanding debt $\tilde{B}$

- The government budget constraint equals

$$\int_0^1 \tau_t \left(w_t s_{i,t} h_{i,t}\right)^{1-\gamma} di = (r_t - 1)\tilde{B}\,.$$

▸ Closing the Model

Calibration

- Inflation target is 2% (annualized). Time discount factor implies a 1% real rate in the DSS.
- Volatility of demand shock reproduces a 10% ZLB frequency.
  Coibion, Dordal-i Carreras, Gorodnichenko, and Wieland (2016)
- Idiosyncratic risk calibrated to match:
  - 30% share of borrowers
    Kaplan, Violante, and Weidner (2014)
  - 10% average marginal propensity to consume
    Johnson, Parker, and Souleles (2006), Parker, Souleles, Johnson, and McClelland (2013)
- Aggregate liquid wealth (i.e., government debt) equals 25% of annual GDP
  McKay, Nakamura, and Steinsson (2016), Kaplan, Moll, and Violante (2018)
- Tax progressivity is $\gamma = 0.18$
  Heathcote, Storesletten, and Violante (2017)
- Rotemberg cost is such that its equivalent Calvo parameter yields a 1-year price duration

More on Calibration

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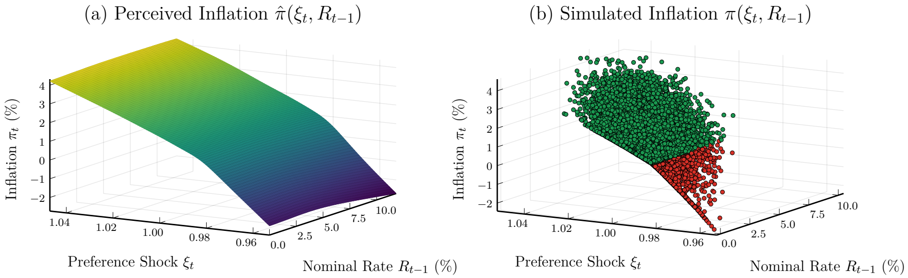
Solution Approach

- Agents form expectations keeping track of how the distribution of bonds evolves
  - Computationally intractable
  - Possible solution: Bounded rationality as in Krusell and Smith (1998)
  - However, this approach hinges on log-linear law of motion

- We use neural networks to determine the fully non-linear laws of motion
  Fernández-Villaverde, Hurtado, and Nuño (2020)
  - In our case, agents predict inflation, $\log \pi_t$, and a term related to inflation expectations, $\log\left(\frac{\pi_{t+1}}{\tilde{\pi}}\right)\frac{Y_{t+1}}{Y_t}$, from the NK Philips curve
  - ZLB introduces non-linearities into the aggregate law of motion
  - Neural Network is able to capture this non-linearity

More on Algorithm

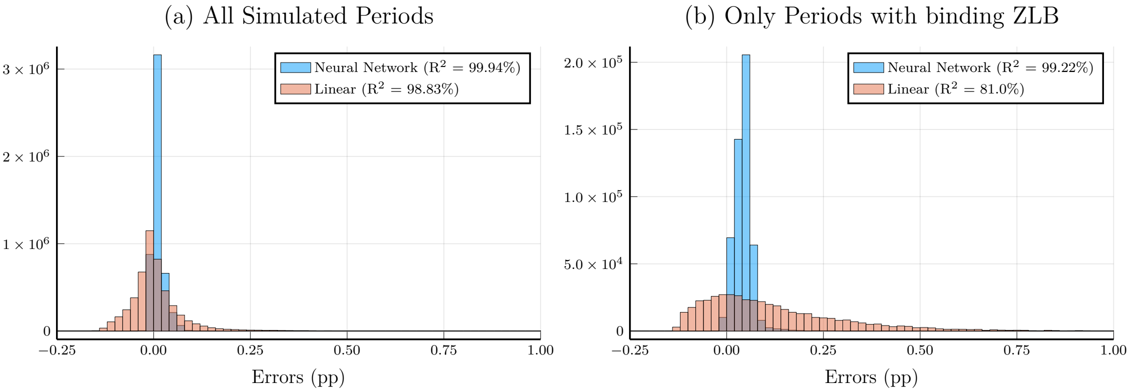
Non-Linearity due to the ZLB



- Different inflation policies arise from bounded rationality assumption
  - Perceived inflation → how agents nowcast inflation
  - Simulated inflation → actual realization of inflation
- ZLB introduces non-linearities into the inflation policies, which are captured by the neural network

More on Algorithm

Nowcast Errors for Inflation



- Neural Network improves upon the linear regression approach, especially at the ZLB
- Results are very similar for forecasts of the inflation expectation term

▸ More on Algorithm

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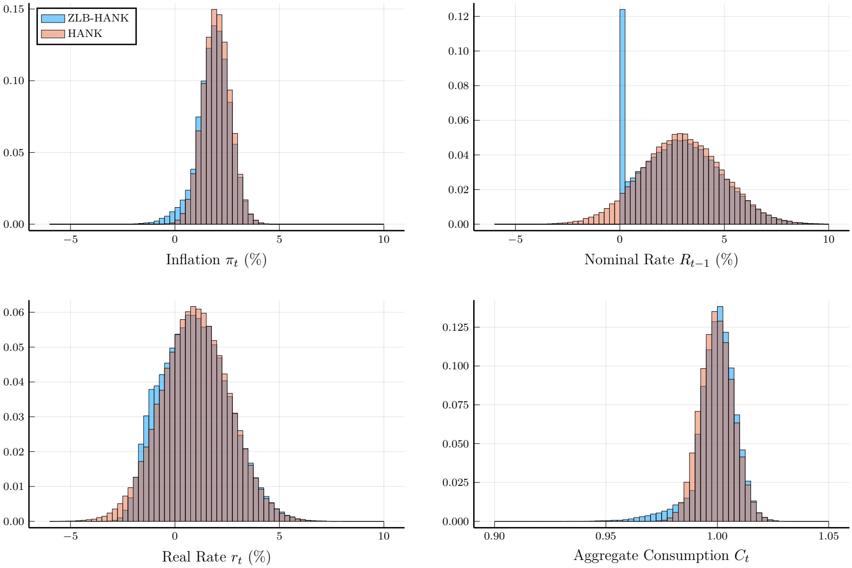
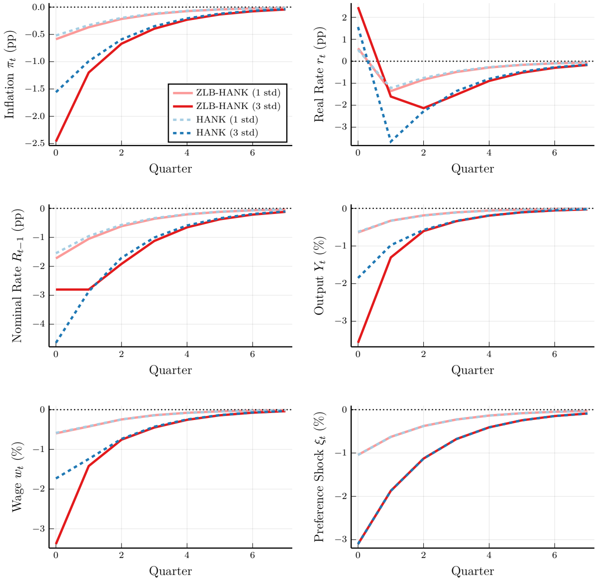
The Macro of the ZLB: Ergodic Distribution



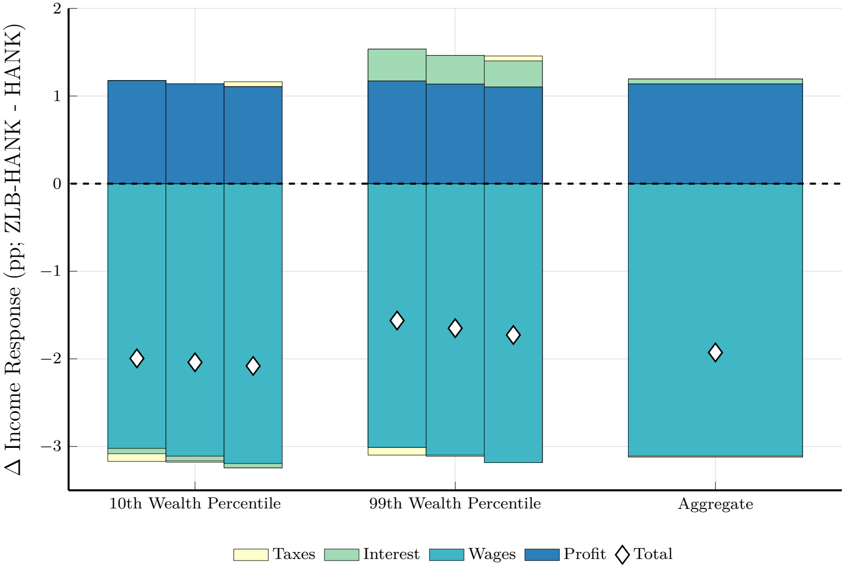
The Macro of the ZLB: Aggregate IRFs



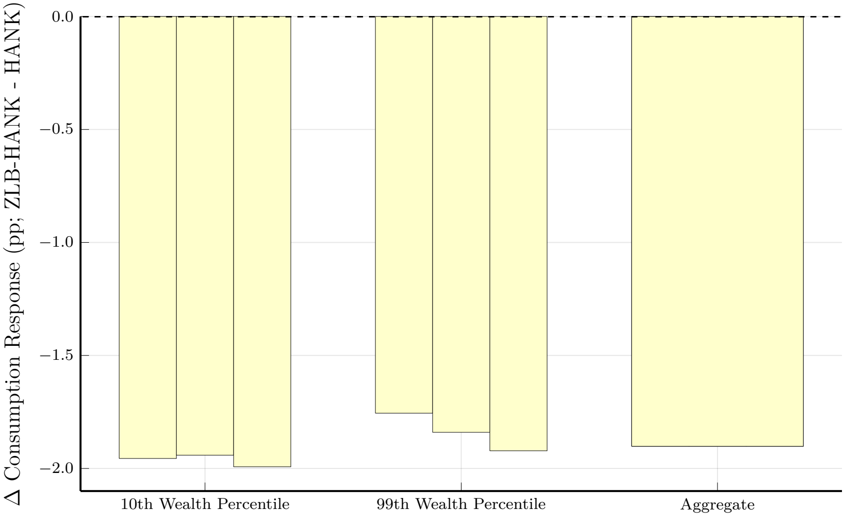
The Macro of the ZLB: Taking Stock

- ZLB skews the dynamics of the model to the left relative to a standard HANK
  - These are the cases in which the nominal rate is constrained by the ZLB
  - Sharp drop in aggregate consumption amidst a deflationary spiral
  - All these dynamics are absent in the standard HANK model
- IRFs to small demand shocks coincide both in the model with and without ZLB
- IRFs to large shocks do differ
  - A large shock brings the nominal interest rate down to zero
  - Much larger drop in both inflation and output
- ZLB events are characterized by deflation and large consumption losses

The Micro of the ZLB: Households' Income IRFs



The Micro of the ZLB: Households' Consumption IRFs



The Micro of the ZLB: Taking Stock

- ZLB alters the distributional effects of a recessionary shocks
- ZLB amplifies the drop in total income
  - This holds for any realization of labor earnings and any position in wealth distribution
  - ZLB makes wages to drop more whereas interest payments relatively rise
  - Larger drop in the total income of wealth-poor households
- ZLB also amplifies the drop in consumption
  - This drop is larger for wealth-poor households
  - Consumption drop for wealth-poor individuals increases by 0.2 pp due to the ZLB
- Burden of recessions tilted towards households at lower end of the wealth distribution

Deterministic and Stochastic Steady States

- What is the difference between the deterministic and stochastic steady states?
  - Deterministic Steady State (DSS): Agents ignore aggregate risks ($\sigma_\xi = 0$)
  - Stochastic Steady State (SSS): Agents make their decisions taking into account aggregate risks ($\sigma_\xi > 0$) but no shock arrives along the equilibrium path
  - Idiosyncratic shocks are taken into account by agents in both cases
- In DSS households do not anticipate the effect of future aggregate shocks, and this case is often referred to as the perfect foresight equilibrium
- Instead, in SSS households are aware of the existence of future aggregate shocks that may hit the economy

Comparison of DSS and SSS in ZLB-HANK, HANK, and ZLB-RANK

	ZLB-HANK		HANK		ZLB-RANK	
Variable	DSS	SSS	DSS	SSS	DSS	SSS
Inflation	2.0%	1.91%	2.0%	1.99%	2.0%	1.93%
Nominal Rate	3.0%	2.80%	3.0%	2.96%	3.22%	3.08%
Real Rate	1.0%	0.89%	1.0%	0.97%	1.22%	1.15%
(Shadow) ZLB Frequency	-	10.17%	-	(6.09%)	-	8.35%
(Shadow) ZLB Duration Quarters	-	1.65	-	(1.50)	-	1.60

Decomposition Exercise

	Real Rate	Nominal Rate	Inflation
ZLB-RANK DSS	1.22%	3.22%	2.0%
ZLB-RANK SSS	1.15%	3.08%	1.93%
(i) Deflationary Bias	0.08pp	0.14pp	0.07pp
ZLB-RANK DSS	1.22%	3.22%	2.0%
ZLB-HANK DSS	1.0%	3.0%	2.0%
(ii) Precautionary Savings - Idiosyncratic Risk	0.22pp	0.22pp	0.0pp
ZLB-RANK DSS	1.22%	3.22%	2.0%
ZLB-HANK SSS	0.89%	2.8%	1.91%
(iii) Total	0.33pp	0.42pp	0.09pp
(iii)-(i)-(ii) Precautionary Savings - Aggregate Risk	0.03pp	0.05pp	0.02pp

The Determinants of the Differences between DSS and SSS Real Rates

- Deflationary bias reduces the level of real rate by 8 bps
- Precautionary savings due to idiosyncratic risk reduce the level of real rate by 22 bps
  - Although this traces back to Aiyagari (1994), our setting grants it a novel perspective
  - Precautionary savings reduce the room of manoeuvre for the central bank's policy rate
  - In standard HANK literature, the precautionary savings are immaterial for aggregate dynamics because of the lack of the ZLB
- Precautionary savings due to aggregate risk reduce the level of real rate by 3 bps

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The Role of the Inflation Target

- Changes in inflation target $\tilde{\pi}$ alter the ZLB frequency and households' expectations $\rightarrow$ affect the level of real interest rates

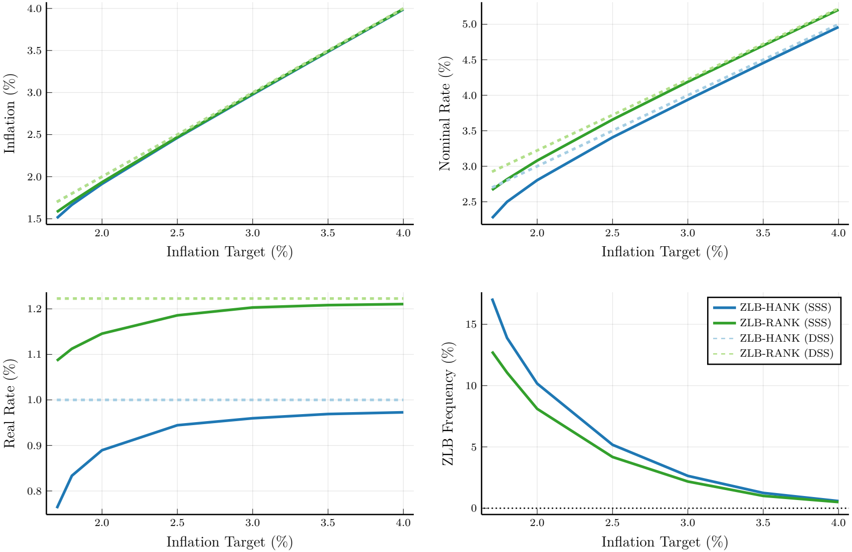
- Monetary policy is not neutral: SSS real rate depends on central bank's inflation target

- The model features a long-run Fisher equation

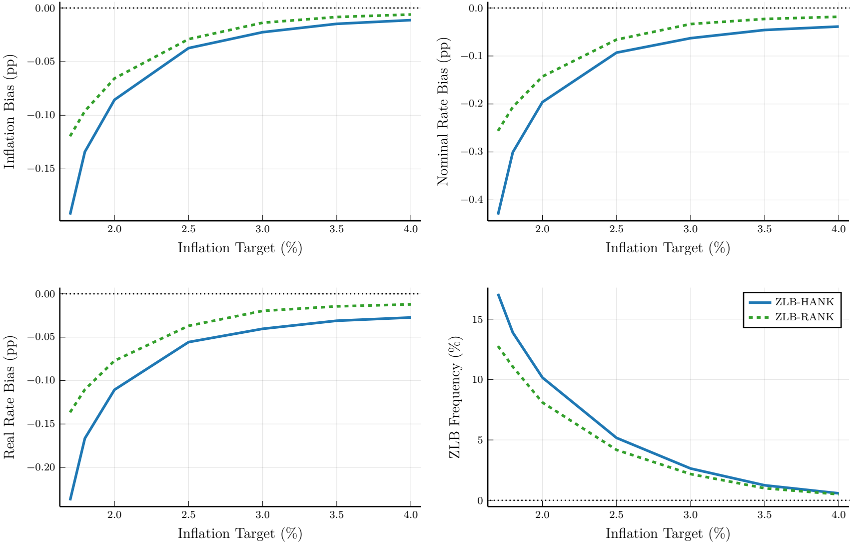
$$i\left(\tilde{\pi}\right) = r\left(\tilde{\pi}\right) + \pi\left(\tilde{\pi}\right), \qquad \text{where } dr/d\tilde{\pi} > 0$$

- To uncover this result, we compare the level of the real interest rate in different model economies, which uniquely differ in the level of the inflation target $\tilde{\pi}$

DSS/SSS in ZLB-RANK/ZLB-HANK as a Function of Inflation Target



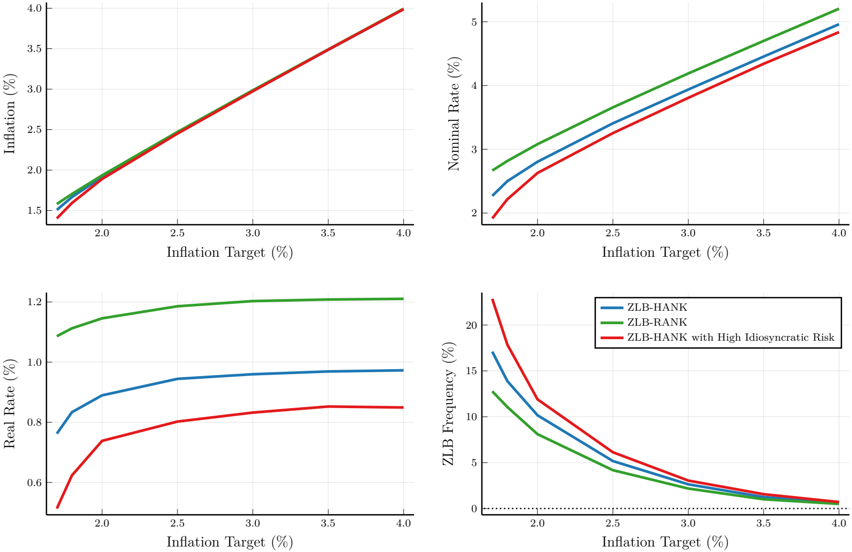
Differences between the SSS and DSS as a Function of Inflation Target



The Monetary Policy Non-Neutrality in the ZLB-HANK Model

- When the inflation target is around 3%, the probability of ZLB events is low $\rightarrow$ not quantitatively relevant in shaping households' expectations
- For targets below 3%, the non-linearity due to the ZLB kicks in $\rightarrow$ SSS and DSS levels diverge
- When the target is 1.7%, the ZLB probability is as high as 20%, and the real rate is 0.75% $\rightarrow$ real rate is 25 bp lower than that associated with the 4% target
- Non-neutrality is also present in RANK model
  Adam and Billi (2007), Nakov (2008), Hills, Nakata, and Schmidt (2019), Bianchi, Melosi, and Rottner (2020)
- However, households' heterogeneity increases substantially the quantitative relevance of the long-run Fisher equation

The Interaction Between the Inflation Target and Wealth Inequality



The Role of Wealth Inequality

- A drop in the inflation target from 4% to 1.7% together with an increase in Gini index of wealth of three p.p. reduces the level of the real rates by 46 bps
- In the RANK model, the drop in the inflation target reduces the real rate by just 14 bps
- The drop in the inflation target is consistent with the reduction in inflation between 1980s-1990s and 2000s-2010s
- The increase in the Gini index of wealth is consistent with that measured by Kuhn and Rios-Rull (2016) in the 2000s
- These two changes accounts for 21% of the overall 150 bps drop in the real rates over the last three decades

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Conclusion

- This paper introduces a HANK model that explicitly incorporates the non-linearity due to the ZLB constraint
- We have solved the model with a novel neural-network algorithm
- The model shows that the ZLB constraint alters the dynamics of both macroeconomic and individual variables
- The burden of recessions is tilted towards wealth-poor individuals
- We uncover the non-neutrality of the central bank's inflation target
  - The model features a long-run Fisher equation
  - Changes in the inflation target reduces the level of the real rate
  - This channel is substantially amplified by households' inequality through changes in precautionary savings

APPENDIX

More on Calibration

	Parameter	Value	Target/Source
		Panel A. Aggregate Risk	
ρ_ξ	AR coefficient of process for ξ	0.6	Bianchi, Melosi & Rottner (2020)
ω_ξ	Standard deviation of ξ shock	0.0105	10% ZLB frequency
		Panel B. Idiosyncratic Risk	
ρ_s	AR coefficient of process for s_t	0.8	10% Average MPC
ω_s	Standard deviation of s_t shock	0.05	30% Borrowers
$\underline{b}$	Borrowing limit	-0.29	Monthly average labor income
		Panel C. Preferences	
β	Discount factor	0.997	1% real interest rate in the DSS
σ	Risk aversion	1	Standard value
$1/\nu$	Frisch elasticity of labor supply	1	Standard value
χ	Disutility of labor	0.71	Labor supply equals 1 in the DSS

More on Calibration

Parameter		Value	Target/Source
		Panel D. Production	
ε	Demand elasticity	7.67	15% price markup
α	Labor share	1	Constant returns to scale
θ	Rotemberg price adjustment cost	79.41	Equivalent to 0.75 Calvo parameter
		Panel E. Monetary Authority	
$\tilde{\pi}$	Inflation Target	$\exp(0.02/4)$	2% Annual inflation target
ϕ_π	Coefficient on inflation	2.5	Standard value
ϕ_y	Coefficient on deviations from steady-state output	0.1	Standard value
		Panel F. Fiscal Authority	
B	Government debt	0.25	Liquid assets = 25% annual GDP
γ	Tax progressivity	0.18	Heathcote, Storesletten & Violante (2017)

Closing the Model

- Labor market clears:

$$\int_0^1 l_{j,t} dj = \int_0^1 s_{it}, h_{i,t} di.$$

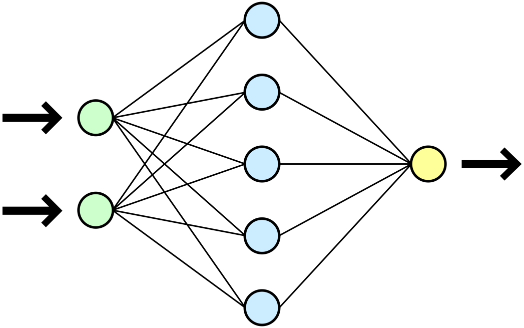
- Bond market clears:

$$\tilde{B} = \int_0^1 b_{i,t} di.$$

- Resource constraint:

$$Y_t = \int_0^1 l_{j,t}^{\alpha} dj = \int_0^1 c_{it} di \, .$$

Neural Networks



- Neural networks are very flexible and can approximate any Borel measurable function
  Fernández-Villaverde, Hurtado, and Nuño (2020)
- In the case for the PLM for inflation π_t we have
  - 2 input nodes ($D = 2$): one for each aggregate state (ξ_t and R_{t-1})
  - 16 hidden nodes ($Q = 16$)
  - 1 output node for the prediction of the neural network (π_t)

Neural Networks

- Mathematically, a Neural Network can be represented as follows

$$h(s;\theta) = \theta_0^1 + \sum_{q=1}^{Q} \theta_q^1 \phi \left(\theta_{0,q}^2 + \sum_{i=1}^{D} \theta_{i,q}^2 s^i \right)$$

  where s is a vector of inputs, θ is a vector of weights and biases, and $\phi = \log(1 + e^x)$ is the activation function

- The weights and biases θ are selected to minimize the loss function

$$\theta^* = \arg\min_{\theta} \frac{1}{2} \sum_{j=1}^{J} \left\| h\left(\mathbf{s}_j;\theta\right) - \widehat{h}_j \right\|^2$$

- The neural network is trained using a back-propagation algorithm with (stochastic) gradient descent

- We can simulate an arbitrary amount of data to train the network